[Gr.G] The general graphics section (20 points)

1. The gamut of a CRT is a triangle on the chromaticity diagram. The vertices of this triangle correspond to the three phosphors of the CRT. CRT phosphors are not pure spectral colors, but are instead mixtures of pure spectral colors. By the superposition principle, mixtures of two or more colors from a set lie strictly within the convex hull of the set. In this case, that set is the set of all pure spectral colors, a.k.a. the spectral locus. Hence, CRT phosphors (and the gamut they define) lie strictly within the spectral locus.

2.

a. Averaging together a block of pixels is postfiltering. Postfiltering cannot eliminate aliasing. The result in this case would be blurry jaggies.

b. Averaging down a higher resolution drawing is supersampling -> reconstruction -> prefiltering (for the pixel rate) -> resampling (at the pixel rate). It reduces aliasing, but it cannot complete eliminate jaggies from lines, since lines contain infinite frequencies and will therefore alias at any sampling rate.

c. Computing areas in this way is analytic prefiltering, also known as unweighted area sampling. For lines of finite width, it the convolution of the step functions representing the line with a rect function representing the prefilter. Since the rect function is not an ideal prefilter (in the spatial domain), this method does not eliminate all aliasing. However, it performs better than supersampling in some situations.

d. This is an example of stochastic sampling, also known as nonuniform sampling. Choosing one point randomly within abutting pixel squares is known as uniform jittered sampling. Stochastic sampling trades aliasing for noise, taking advantage of the human visual system's relatively higher tolerance for noise. Drawing one random sample per pixel will yield a very noisy image.

3. One per set of parallel lines, hence infinitely many. Most have fewer. Three is typical for architectural environments, because buildings typically contain only three directions of lines, corresponding to the three Cartesion axes.

[Gr.S] The specialized graphics section (40 points)

1. It migrates them along curves toward the primaries, thus shifting their hue and increasing their saturation. An example:

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formula example
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stored value: v (.25,.5,1)
corrected: c = v^(1/gamma) (.5..707,1)
displayed: d = c^gamma = v (.25,.5,1)
w/o correction: d2 = v^gamma (.125,.25,1)
```

d2 is more saturated and bluer than d, as can be verified by converting these RGB values to HSV. Note that the failure affects only colors having values between 0 and 1, so it would not affect the primaries themselves, e.g. (0,0,1).

2.

a. Aliasing is high frequencies masquerading as low frequencies due to undersampling. In the frequency domain, aliasing occurs when replicas of the baseband spectrum, which are introduced as a direct consequence of sampling, overlap one another, thus contaminating the baseband spectrum.

Lines are delta sheets and polygon edges are step functions; both contain infinite frequencies. Therefore, any sampling of lines or edges without prior prefiltering to eliminate high frequences will exhibit aliasing. The low-frequency rhythm of notches in a jagged line or edge are examples of this aliasing.

b. Reconstruction is the process of interpolating between samples to regenerate a continuous function from its samples. It is equivalently described as convolving the samples by a filter function, often called the postfilter. The ideal postfilter is a sinc function. In the frequency domain, reconstruction is the process of isolating the baseband spectrum from its replicas by windowing. Windowing (multiplication) is the frequency domain counterpart of convolution in the spatial domain, and the frequency domain counterpart of of the sinc function is a rect function.

Failure to properly reconstruct samples of lines or edges leads to an error called rastering, which is the presence in the reconstruction of residual energy at the sampling rate. Overfocused CRTs exhibit rastering. Rastering is distinct from aliasing, and it does not cause jaggies. However, the square pixels on LCD displays, which correspond to a rect reconstruction filter, may make jaggies that are already present in a sampled image more evident.

c. Quantization partitions the range of an image into discrete steps, as opposed to sampling, which in some sense partitions its domain. Excessive quantization of smooth images leads to the appearance of contours. These contours may be jagged, but they are not usually called jaggies. In the frequency domain, these contours constitute false high frequencies, but they are not introduced by overlapping spectra as with aliasing.

3.
a. The probability of one ray hitting the light is proportional to the solid angle omega subtended by the projection of the spotlight as a fraction of the solid angle of the hemisphere above the point x. Because we are given that h is large, we can approximate the solid angle as (A cos psi)/r^2 where A, the area of the spotlight, is pi (d/2)^2, psi, the inclination of the spotlight with respect to a line connecting its center and the point x, is atan(w/h), and r, the length of that line, is w/cos(psi). So, since the solid angle of the hemisphere is 2 pi steradians:

```
prob(1 ray / 1 hit) = (A cos psi)/(2 pi r^2)

prob(m rays / no hits) = [1 - (A cos psi)/(2 pi r^2)]^m
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prob(m rays / any hits) = 1 - [1 - (A cos psi)/(2 pi r^2)]^m
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b. Intuitively, the probability of a total miss is highest in the corners, since both the angle of projection and the distance to the light are maximized at these points. Using the result from (a):

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w = sqrt(2(2.5^2)) = 3.54

r = sqrt(3(2.5^2)) = 4.33

A = pi (0.5)^2 = 0.00785

psi = atan(sqrt(2)) = 54.7 degrees

prob(50 rays / no hits) = 0.9981
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c. Since you know that the spotlight is an emitter, cast more rays towards it than elsewhere. This is an example of importance sampling.