AI Comp 2003 solutions (by CS PhD 2006)

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1 Search

a) $2^{k+1} - 1$
b) $2^{d+1} - 2^{d-k+1} + 1$ (complete tree except descendants of the solution node)
c) $2^{k+2} - k - 3$. We can get this from summing up the cost of breadth-first search with max depth = 0, 1, ..., k.

$$\begin{aligned}
\sum_{i=0}^{k} \text{complete tree of depth i} &= \sum_{i=0}^{k} 2^{i+1} - 1 && (1)\\
&= \left(\sum_{i=0}^{k+1} 2^i\right) - 1 - \sum_{i=0}^{k} 1 && (2)\\
&= 2^{k+2} - 1 - 1 - (k+1) && (3)\\
&= 2^{k+2} - k - 3 && (4)
\end{aligned}$$

2 Logic

a) True.
b) False. $\phi = p \vee q$. $\Gamma = \{p\}$. $\Delta = \{q\}$
c) True.
d) True.
e) True. Tautology

3 Automated Reasoning

1. $\neg\forall x(p(x) \Rightarrow r(x))$ (negate the conclusion and add to our premises)
2. $\forall x(p(x) \Rightarrow \exists y(q(x,y) \vee q(y,x)))$ (premise)
3. $\forall x\forall y((q(x,y) \vee q(y,x)) \Rightarrow r(x))$ (premise)
4. $\neg\forall x\neg p(x) \vee r(x)$ (eliminate implication from 1)
5. $\forall x(\neg p(x) \vee \exists y(q(x,y) \vee q(y,x)))$ (eliminate implication from 2)
6. $\forall x\forall y(\neg(q(x,y) \vee q(y,x)) \vee r(x))$ (eliminate implication from 3)
7. $\exists x(p(x) \wedge \neg r(x))$ (push negation inside, 4)
8. $\forall x\forall y((\neg q(x,y) \wedge \neg q(y,x)) \vee r(x))$ (push negation inside, 6)
9. $\neg p(x) \vee (q(x, f(x)) \vee q(f(x), x))$ (skolemize, 5)

10. $(p(C) \wedge \neg r(C))$ (skolemize, 7)
11. $((\neg q(x, f(x)) \wedge \neg q(f(x), x)) \vee r(x))$ (skolemize, 8)
12. $(\neg q(x, f(x)) \vee r(x)) \wedge (\neg q(f(x), x) \vee r(x))$ (distribute $\vee$, 11)
13. $p(C)$ (split up $\wedge$, 10)
14. $\neg r(C)$ (split up $\wedge$, 10)
15. $(\neg q(x, f(x)) \vee r(x))$ (split up $\wedge$, 12)
16. $(\neg q(f(x), x) \vee r(x))$ (split up $\wedge$, 12)
17. $\neg q(C, f(C))$ (resolving 15 and 14 by setting x to C)
18. $\neg q(f(C), C)$ (resolving 16 and 14 by setting x to C)
19. $q(C, f(C)) \vee q(f(C), C)$ (resolving 9 and 13 by setting x to C)
20. $q(f(C), C)$ (resolving 19 and 17)
21. $\perp$(false) (resolving 18 and 20)
22. $\forall x(p(x) \Rightarrow r(x))$ (Negation of one of the premises is true)

4 Probability

a) $1/\binom{6}{3}$
b)

$$p(d_2 = g|d_1 = g) = \frac{p(d_2 = g, d_1 = g)}{p(d_1 = g)} \quad (5)$$

$$= \frac{1/3}{1/3 \cdot 1 + 1/3 \cdot 1/2} \quad (6)$$

$$= \frac{2}{3} \quad (7)$$

5 Learning

a) There are two answers. One makes "a" the root node and "d" the child nodes. But you can switch them and make "d" the root node. The tree has depth 1 (beginning with depth 0).
b) Since the probability that "Goal" is 1 is $\frac{1}{2}$, $p = \frac{1}{2}$ and $n = \frac{1}{2}$. Therefore, $-(p * logp) - (n * logn) = 1$.
c) If a is 1, the probability that Goal is 1 is $\frac{1}{2}$. If a is 0, the probability that Goal is 1 is $\frac{1}{2}$. Therefore, information needed is 1 for both cases.
d) Info gain from a is 0. Since the tree correctly classifies all examples, information gain from d given a is 1 - (info gain from a) = 1.