Computer Science Department
Stanford University
Comprehensive Examination in LOGIC

Autumn 1994

18 October 1994

READ THIS FIRST!

1. You should mark your answers only in the ANSWER SHEET that is provided with the this part of the Comprehensive Examination. Be sure to write your MAGIC NUMBER on the answer sheet.
2. This exam is OPEN BOOK.
3. There are 5 pages of the exam including the answer sheet.
4. The exam takes 30 minutes and there are 18 questions in the exam. You will receive **1** points for each CORRECT answer, **0** points for not answering, and **-1/3** points for each WRONG answer. Mark "X" in at most one box for each question. If you mark in more than one choices, you will receive **-1** points for that question.

   Example:

	(A)	(B)	(C)	(D)
1.			X	

   If you decide to change your answer, erase the old marks completely or cross out with two lines, "~~X~~".

   Example:

	(A)	(B)	(C)	(D)
1.	X		~~X~~	

ANSWER SHEET
Comprehensive Examination in LOGIC
Autumn 1994

MAGIC NUMBER: ______

	(A)	(B)	(C)	(D)
1.				
2.				
3.				
4.				
5.				
6.				
7.				
8.				
9.				
10.				

	(A)	(B)	(C)	(D)
11.				
12.				
13.				
14.				
15.				
16.				
17.				

Problem 1:

If P is a necessary condition for Q, then

(A) Q is a sufficient condition for P

(B) $\neg Q$ is a sufficient condition for $\neg P$

(C) $\neg Q$ is a sufficient condition for P

(D) P is a sufficient condition for Q

Problem 2:

The sentence *"Paul goes to the store only if Mary drinks milk"* is best represented by which of the following (where P represents *"Paul goes to the store"* and M represents *"Mary drinks milk"*):

(A) $\neg M \rightarrow \neg P$

(B) $M \rightarrow P$

(C) $M \rightarrow \neg P$

(D) $\neg M \rightarrow P$

Problem 3:

Inductively define the formulas ϕ_n, for all $n > 0$, as follows:

$\phi_1 : \quad p$

$\phi_2 : \quad (p \rightarrow p)$

$\phi_n : \quad (\phi_{n-1} \rightarrow p)$

For which n is the formula ϕ_n a tautology?

(A) For all n

(B) For all odd n

(C) For all even n

(D) None of the above

Problem 4:

A set of formulas, $\{\sigma_1, \sigma_2, \ldots, \sigma_n\}$, is not consistent, if and only if:

(A) the sentence $\neg\sigma_1 \wedge \neg\sigma_2 \wedge \ldots \wedge \neg\sigma_n$ is satisfiable.

(B) the sentence $\neg\sigma_1 \vee \neg\sigma_2 \vee \ldots \vee \neg\sigma_n$ is satisfiable.

(C) the sentence $\neg\sigma_1 \wedge \neg\sigma_2 \wedge \ldots \wedge \neg\sigma_n$ is valid.

(D) the sentence $\neg\sigma_1 \vee \neg\sigma_2 \vee \ldots \vee \neg\sigma_n$ is valid.

Problem 5:

Consider the sentences

1. $P \rightarrow \neg P$
2. $(P \rightarrow \neg P) \vee (\neg P \rightarrow P)$

(A) 1 is unsatisfiable, and 2 is valid.

(B) 1 is satisfiable, but not valid; and 2 is valid.

(C) both 1 and 2 are satisfiable, but neither is valid.

(D) 1 is unsatisfiable, and 2 is satisfiable but not valid.

Problem 6:

Suppose $\mathcal{F}$ is satisfiable, then it necessarily follows that

(A) $\mathcal{F}$ is valid

(B) $\neg\mathcal{F}$ is valid

(C) $\neg\mathcal{F}$ is not satisfiable

(D) none of the above

Problem 7:

Let $A_1, \ldots, A_n$ be the axioms of a theory $\mathcal{T}$. Let S be the set of sentences that are valid in $\mathcal{T}$, and let M be the set of models of $\mathcal{T}$.

Consider the theory $\mathcal{T}'$, defined by dropping one of the axioms of $\mathcal{T}$, with set of valid sentences S' and set of models M'. Which of the following is true?

(A) $S' \subseteq S$ and $M' \subseteq M$

(B) $S' \subseteq S$ and $M \subseteq M'$

(C) $S \subseteq S'$ and $M' \subseteq M$

(D) $S \subseteq S'$ and $M \subseteq M'$

Problem 8:

Assume we have a set A of axioms, and let $\mathcal{T} = \text{Cn } A$ be the theory consisting of the consequences of A. If A is inconsistent, then:

(A) $\mathcal{T}$ contains only A.

(B) $\mathcal{T}$ contains only formulas valid in all models.

(C) $\mathcal{T}$ is empty.

(D) $\mathcal{T}$ contains all formulas.

Problem 9:

Suppose we have a deductive tableau proof of an initial tableau consisting of the goal $\mathcal{F}[x]$ whose only free variable is x. In this proof, assume that the variable x is never replaced by any other variable or term by application of a substitution.

Consider the following statements:

1. $(\exists x)\mathcal{F}(x)$
2. $\mathcal{F}[a]$, where a is a new constant (not occurring in the tableau)
3. $(\forall x)\mathcal{F}(x)$

Which of the above statements are valid?

(A) none

(B) 1 only

(C) 1 and 2 only

(D) 1,2 and 3

Problem 10:

If a sentence $\mathcal{E}$ has strictly positive polarity in $\mathcal{F}[\mathcal{E}^+]$ and $\mathcal{E}$ is true under interpretation I, then which of the following holds:

1. $\mathcal{F}[true]$ is true under I
2. $\mathcal{F}[false]$ is false under I

(A) 1 only

(B) 2 only

(C) both 1 and 2

(D) neither

Problem 11:

Which of the following are unifiable?

1. $f(x,x)$
2. $f(g(x),y)$
3. $f(z,f(z))$

(A) (1,2) and (1,3) only

(B) (2,3) only

(C) (1,2) and (2,3) only

(D) none

Problem 12:

Consider the following sentence:

$$(\exists y)(\forall z)p(y,z) \rightarrow (\exists z)(\forall y)p(z,y)$$

Which of the following skolemizations preserves validity?

(A) $p(a,z) \rightarrow p(y,f(y))$

(B) $p(a,z) \rightarrow p(z,f(z))$

(C) $p(y,g(y)) \rightarrow p(z,f(z))$

(D) $p(a,z) \rightarrow p(y,b)$

Problem 13:

In first-order logic, suppose ϕ logically follows from an infinite set of sentences Δ, then

(A) ϕ is derivable from some finite subset of Δ

(B) ϕ is derivable from all finite subsets of Δ

(C) ϕ is derivable from every infinite set of sentences

(D) none of the above

Problem 14:

Determining validity of a sentence ϕ in a mathematical theory with a sound and complete deductive system is: *(Note: decidable = recursive, semi-decidable = recursively enumerable.)*

(A) always decidable

(B) always semi-decidable

(C) always undecidable

(D) none of the above

Problem 15:

Which of the following are true?

I. It is possible to give a decision procedure for the validity problem of **propositional logic**.

II. It is possible to give a decision procedure for the validity problem of **predicate logic**.

III. It is possible to give an algorithm for predicate logic such that given any sentence of **predicate logic**, the algorithm returns true precisely when the sentence is valid, and either returns false or does not terminate when the statement is not valid.

(A) Only I and II

(B) I, II, and III

(C) Only I and III

(D) Only III

Problem 16:

Which of the following are true?

I. The theory of the natural numbers with addition and multiplication is inconsistent.

II. The theory of the natural numbers with addition and multiplication has no sound and complete axiomatization.

III. The theory of the natural numbers with addition, but no multiplication has a sound and complete axiomatization.

(A) Only I

(B) Only II and III

(C) Only II

(D) Only III

Problem 17:

Which of the following connectives forms a propositionally complete set, i.e., such that all the other propositional connectives can be expressed in terms of them? Assume that the symbols *true* and *false* cannot be used, except *false* in 2.

1. $\rightarrow$
2. $\rightarrow$, *false*
3. $\vee$, $\wedge$, $\leftrightarrow$
4. $\neg$, $\leftrightarrow$
5. $\neg$, $\rightarrow$
6. *if ... then ... else*
7. *nand*

(A) 2, 4, 5, 6, 7.

(B) 3, 5, 7.

(C) 2, 5, 7.

(D) 1, 2, 4, 5.